Reply to "Comment on 'Localized vortices with a semi-integer charge in nonlinear dynamical lattices' "

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We reply to the Comment by M. Johansson [Phys. Rev. E **66**, 048601 (2002)]. In particular, we point out that the one-dimensional solutions with the semi-integer vorticity were found in the original paper with a finite accuracy, which, in fact, is in agreement with a figure from the Comment. Moreover, to highlight the physical relevance of such solutions, we have additionally performed direct simulations which clearly demonstrate that these solutions persist for *extremely long* times.

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In the Comment [1], Johansson criticizes solutions describing one- and two-dimensional (1D and 2D) states with the semi-integer vorticity (topological charge) $S=1/2$ in the discrete nonlinear Schrödinger equation, which were reported in our paper [2]. The main point in the criticism is that the solutions cannot exist as exact ones, due to the presence of a nonzero current in the 1D case. The Comment also alludes to similar arguments in the 2D case, even though a 2D version of the current was not produced and a rigorous proof was not provided.

As concerns the 1D case, we agree that the existence of a rigorous $S=1/2$ stationary DNLS solution is ruled out by the argument presented in the Comment (the presence of the current), and Ref. [2] had indeed missed this point. However, it is relevant to stress that the solutions presented in Ref. [2] had been found with a *finite* accuracy (as was explicitly stated in the paper). It is straightforward to see that the accuracy of the 1D numerical solutions reported in Ref. [2] is the same as in the Comment [1]; see, for instance, Fig. 1 of Ref. [1]. It is clear then from the results presented in *both* Refs. [1] and [2] that such solutions will be (dynamically) destroyed, due to the fact that they are not exact ones, at times which are extremely long ($\sim 10^8$–10^{10} or larger, in units used both in Refs. [2] and [1]). So large time (or propagation distance, in the case of the most relevant application to an array of nonlinear optical waveguides [3]) is irrelevant to any experimental situation, which means that the $S=1/2$ solution may be observed in a 1D dynamical lattice.

Notice that the two statements above (the nonexactness of the solution, which causes its destruction over the extremely long time scale, and the finite accuracy, about which a caveat was made in Ref. [2]) cannot be distinguished in dynamical simulations. It is useful to know that such solutions are not rigorously exact, as is shown in the Comment, but they *are* metastable extremely long-lived states, relevant to the time or distance of any experimental observation possible.

To further illustrate the above statements, we have performed long dynamical simulations (involving 1000 dynamical steps or more) in which we monitored the evolution of the $S=1/2$ solution reported in Refs. [1,2] (for $C=0.001$, and with the $\Delta\phi=\pi/2$ phase shift between the lattice sites $n=45$ and $n=55$, which corresponds to $S=1/2$). It is obvious from Fig. 1 that, for the very long dynamical evolution reported here, the two sites remain $\pi/2$ out of phase, with $\Delta\phi$ deviating from this value by roughly 10^{-10} by the end of this long simulation.

[1] M. Johansson, preceding paper, Phys. Rev. E **66**, 048601 (2002).
[2] P.G. Kevrekidis, B.A. Malomed, A.R. Bishop, and D.J. Frantzeskakis, Phys. Rev. E **65**, 016605 (2001).
[3] H.S. Eisenberg, Y. Silberberg, R. Morandotti, A.R. Boyd, and J.S. Aitchison, Phys. Rev. Lett. **81**, 3383 (1998).

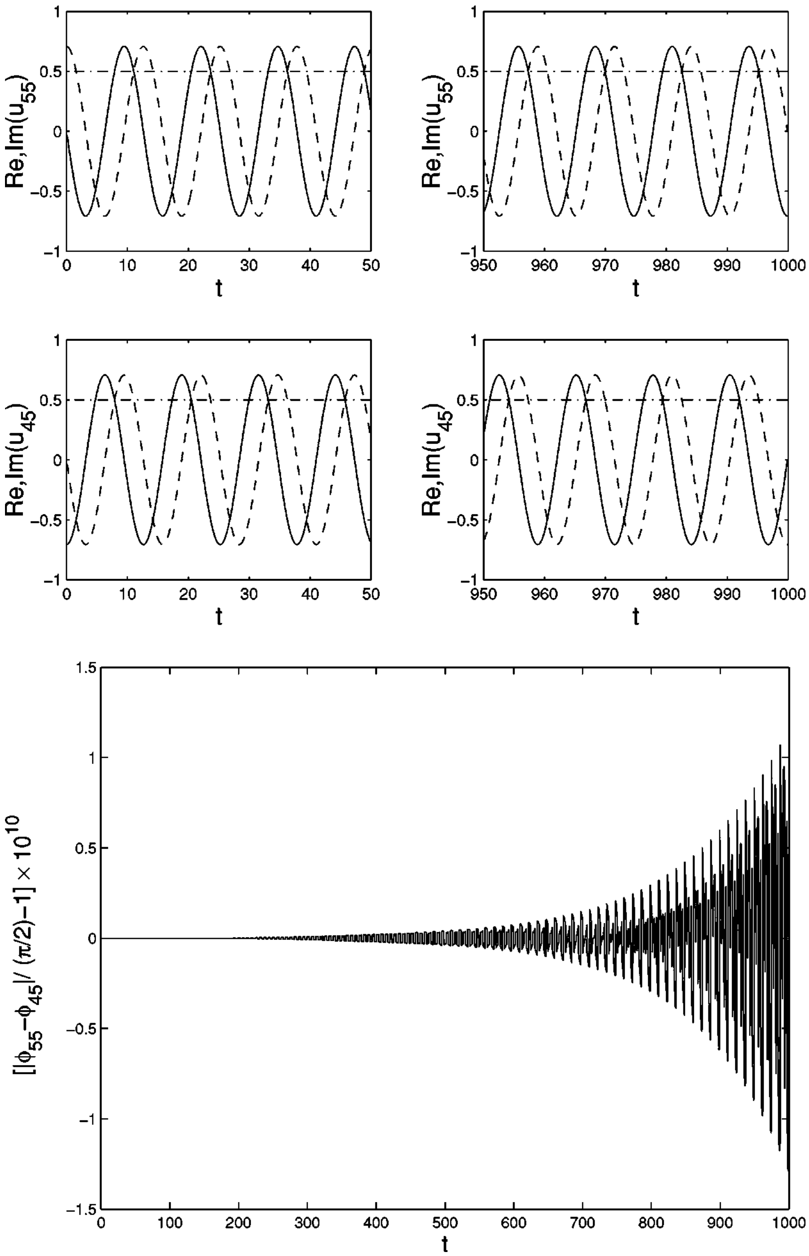

FIG. 1. The upper panel shows the real and imaginary parts of the field at the site $n=55$ in the beginning (times in the interval 0–50) and at the end (times 950–1000) of the long simulation. The solid and dashed lines depict the real and imaginary parts of the field, while the dash-dotted line shows the sum of their squares. Similar findings are shown in the lower panel for the site $n=45$, which is initially (as well as in the end) $\pi/2$ out of phase in comparison with $n=55$. In the bottom subplot, we display the evolution of the phase difference between the two sites (how much it differs from $\pi/2$). It is clear that over the duration of the simulation, it becomes different from $\pi/2$ by less than one part in 10^{10}.